

# Congruence



- ★ Two figures are congruent if one shape lies exactly on top of the other after one or more transformations (translation, reflection, rotation).
- ★ Solve problems using properties of congruent figures, justifying reasoning and making generalisations.
- ★ The minimal conditions for congruence (SSS, SAS, ASA and RHS) and the conditions that do not prescribe congruence (ASS, AAA).
- ★ Plot the vertices of two-dimensional shapes on the Cartesian plane, translating, rotating or reflecting the shape and using coordinates to describe the transformation.

I was born  
to tessellate



## A TASK

The entrance to the library is to be tiled. Design a tessellation that could be used to cover the floor of the entrance.

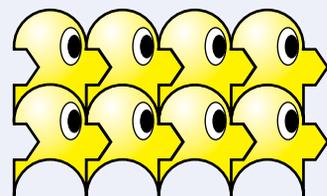
- Research tessellations using basic shapes such as squares, equilateral triangles and even circles.
- Research how MC Escher made tessellations.
- Create your own design.
- Present your design.

## A LITTLE BIT OF HISTORY

- |         |  |
|---------|--|
| 4000 BC | Sumerian tessellations.  |
| 3000 BC | Tessellations in the form of mosaics are used in Mesopotamia.  |
| 1619    | Kepler wrote about regular and semiregular tessellation.   |
| 1891    | Fedorov proved that tiling of the plane is based on 17 different groups of isometries.               |
| 1936    | MC Escher begins to create hundreds of amazing tessellations.  |
| 2008    | Tessellation software allows for realistic visualisation in movies and games (eg. large armies etc). |

Tessellations (to pave) are everywhere in our society from ancient mosaics and architecture to modern art, floors, and quilting. Tessellations also occur in nature from beehives to rock structures.

See Investigation 7.1 to make your own tessellation.



## Congruence

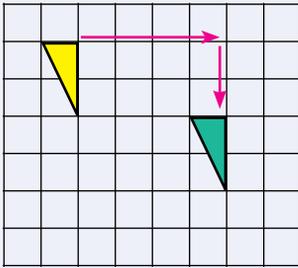
**Congruent shapes** have the same shape and size.  
They fit exactly on top of each other.

The following pairs of shapes are congruent:

1  **and**  
The second shape has been reflected

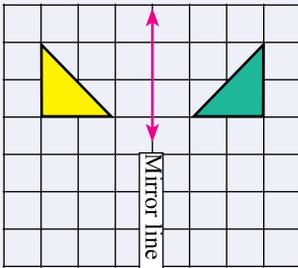
2  **and**  
The second shape has been translated  
and rotated

A **translation** slides a shape across a page.  
A translation produces a congruent shape.



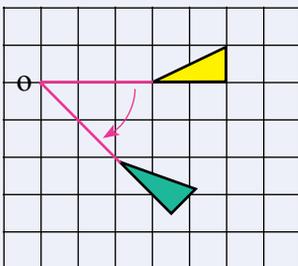
The triangle has been translated  
4 units to the right  
and 2 units down

A **reflection** is a mirror image.  
A reflection produces a congruent shape.



The triangle has been reflected across  
the mirror line

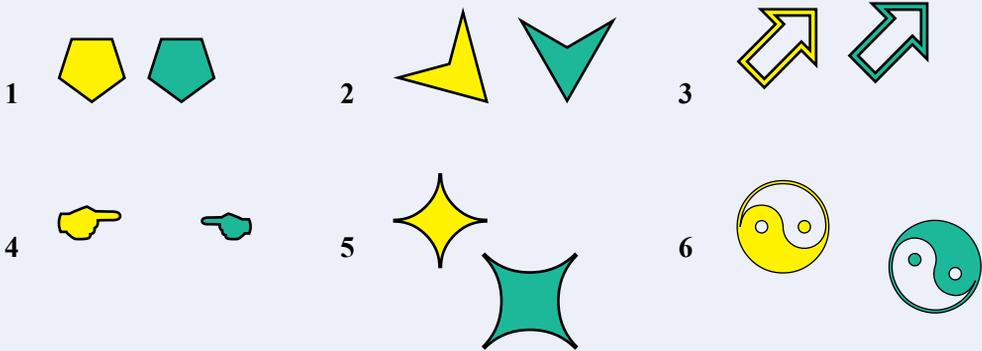
A **rotation** turns a shape about a point.  
A rotation produces a congruent shape.



The triangle has been rotated  $45^\circ$   
around the point O

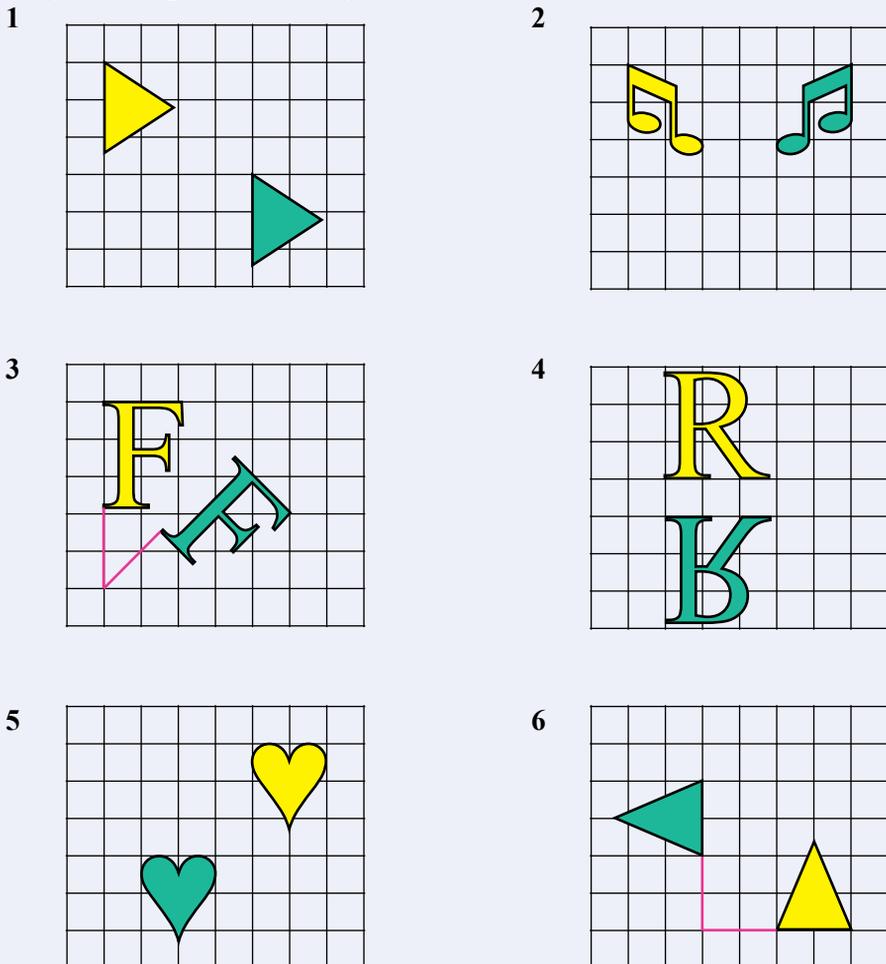
### Exercise 7.1

Which of the following pairs of shapes are congruent (If cut out, they should fit exactly on top of each other)?



### Exercise 7.2

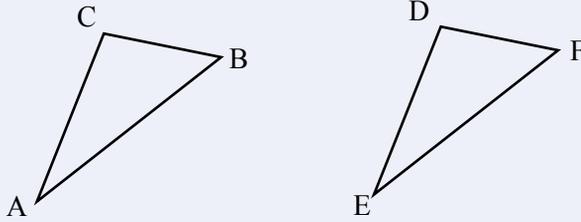
Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:



# Congruent Triangles

**Congruent triangles** have exactly the same shape and size.

The symbol for congruence is  $\equiv$  or  $\cong$



Triangle ABC is congruent to triangle EFD or

$$\triangle ABC \equiv \triangle EFD$$

The **angles must be named** in matching order:

$$\angle A = \angle E$$

$$\angle B = \angle F$$

$$\angle C = \angle D$$

The **sides must be named** in matching order:

$$\text{side AB} = \text{side EF}$$

$$\text{side BC} = \text{side FD}$$

$$\text{side CA} = \text{side DE}$$

### Exercise 7.3

Correctly name the congruent pair of triangles (angles and sides must match):

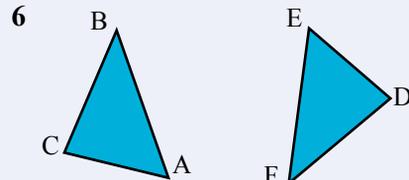
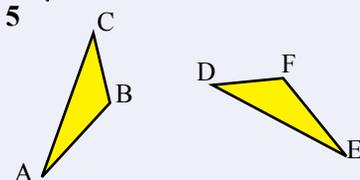
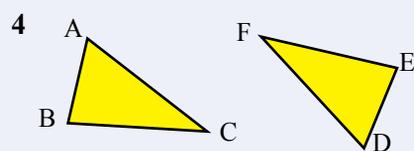
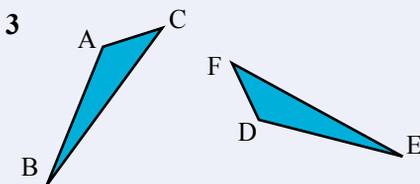
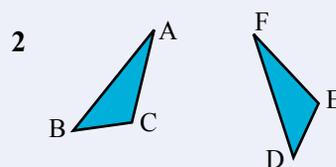
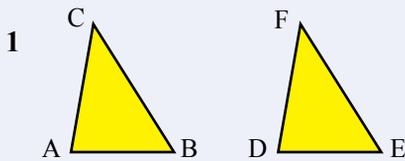
$$\angle A = \angle F$$

$$\angle B = \angle D$$

$$\angle C = \angle E$$

$$\underline{\triangle ABC \equiv \triangle FDE}$$

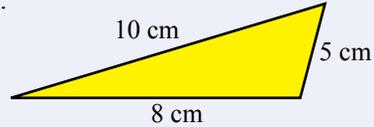
If the angles are in matching order then the sides will be in matching order. And vice versa.



## Tests for Congruent Triangles

**Activity: The SSS test - If the three sides match are they congruent?**

1. Draw a triangle with sides of 5 cm, 8 cm, and 10 cm.
2. Match your triangle with other people in the classroom who have also drawn a triangle with sides of 5 cm, 8 cm, and 10 cm.
3. Are the triangles congruent?

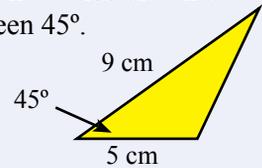


**Activity: The AAA test - If the three angles match are they congruent?**

1. Draw a triangle with angles of  $60^\circ$ ,  $20^\circ$ , and  $100^\circ$ .
2. Match your triangle with other people in the classroom who have also drawn a triangle with angles of  $60^\circ$ ,  $20^\circ$ , and  $100^\circ$ .
3. Are the triangles congruent?

**Activity: The SAS test - If two sides and the angle in between the two sides match are they congruent?**

1. Draw a triangle with sides of 5 cm and 9 cm with the angle in between  $45^\circ$ .
2. Match your triangle with other people in the classroom who have also drawn a triangle with sides of 5 cm and 9 cm with the angle in between  $45^\circ$ .
3. Are the triangles congruent?

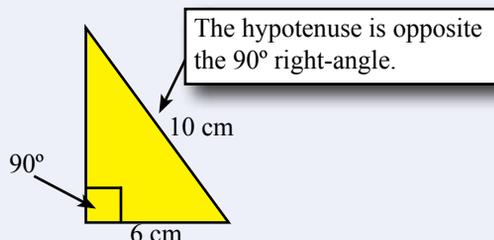


**Activity: The ASA test - If two angles and a side match are they congruent?**

1. Draw a triangle with angles of  $30^\circ$  and  $45^\circ$  and a side of 8 cm.
2. Match your triangle with other people in the classroom who have also drawn a triangle with angles of  $30^\circ$  and  $45^\circ$  and a side of 8 cm.
3. Are the triangles congruent?

**Activity: The RHS test - If a right-angle, the hypotenuse and another side match are they congruent?**

1. Draw a triangle with a right-angle ( $90^\circ$ ), a hypotenuse of 10 cm, and another side of 6 cm.
2. Match your triangle with others who have also drawn a triangle with a right-angle ( $90^\circ$ ) a hypotenuse of 10 cm, and another side of 6 cm.
3. Are the triangles congruent?

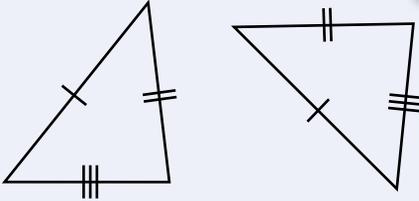


## Tests for Congruent Triangles

The activities on the previous page lead to four conditions that each provide a test of whether two triangles are congruent:

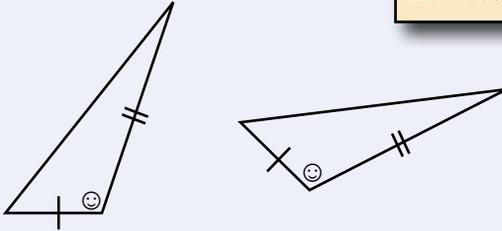
**SSS** (side, side, side).

Two triangles are congruent if the three sides in one triangle are the same length as the matching sides on the other triangle.



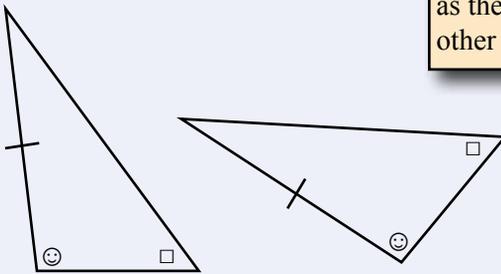
**SAS** (side, angle, side).

Two triangles are congruent if two sides and the included angle in one triangle are the same size as the matching sides and included angle on the other triangle.



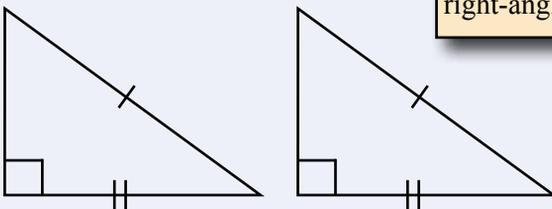
**ASA** (angle, side, angle).

Two triangles are congruent if two angles and a side in one triangle are the same size as the matching two angles and side on the other triangle.



**RHS** (right-angle, hypotenuse, side)

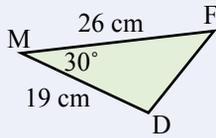
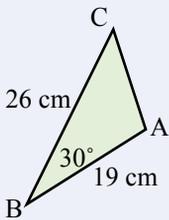
Two triangles are congruent if the hypotenuse and another side in one right-angled triangle are the same size as the matching hypotenuse and another side on the other right-angled triangle.



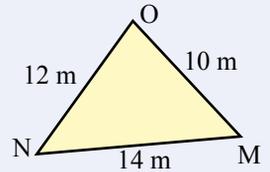
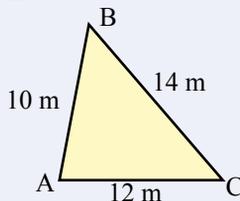
**Exercise 7.4** Use the tests for congruence to test whether the following pairs of triangles are congruent:

		<p>side AC = side WP  <math>\angle C = \angle P</math> {angle inbetween}              side CB = side PT</p> <p><u><math>\triangle ACB \equiv \triangle WPT</math> {SAS}</u></p>
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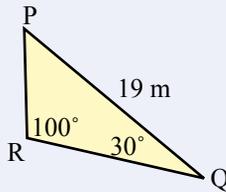
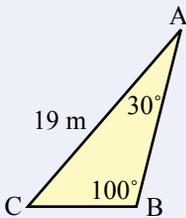
1



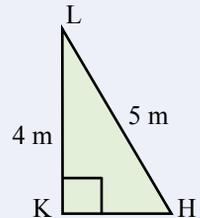
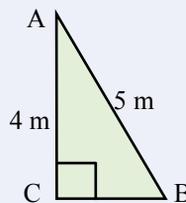
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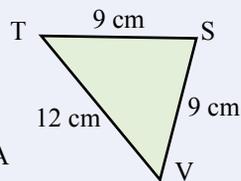
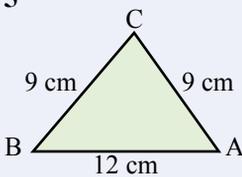
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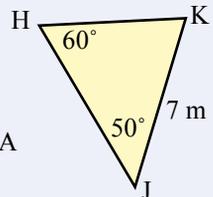
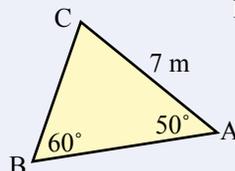
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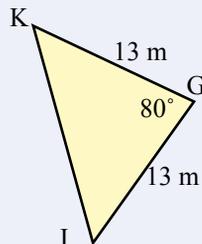
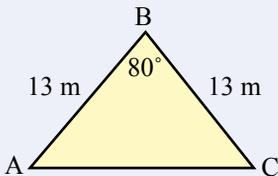
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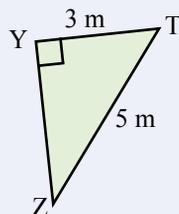
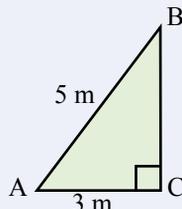
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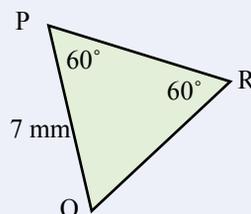
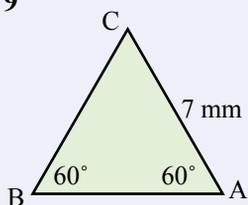
7



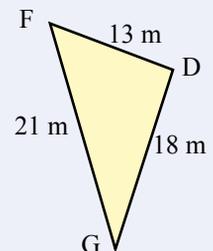
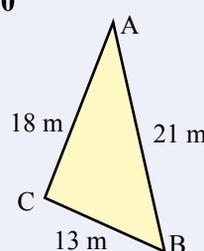
8



9



10



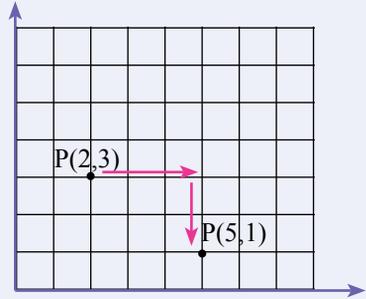
# Transformations

## Exercise 7.5

What are the new coordinates of  $P(2,3)$  after  $P$  is translated 3 units right and 2 units down?

$$= P(2+3, 3-2)$$

$$= \underline{P(5,1)}$$

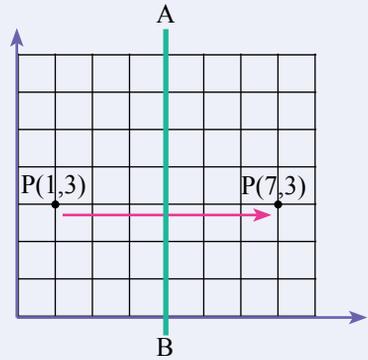


- 1 What are the new coordinates of  $P(1,4)$  after  $P$  is translated 2 units right and 2 units down?
- 2 What are the new coordinates of  $P(2,3)$  after  $P$  is translated 4 units right and 1 unit down?
- 3 What are the new coordinates of  $P(5,4)$  after  $P$  is translated 3 units left and 2 units up?

What are the new coordinates of  $P(1,3)$  after  $P$  is reflected in the line  $AB$ ?

$$= P(1+2 \times 3, 3)$$

$$= \underline{P(7,3)}$$

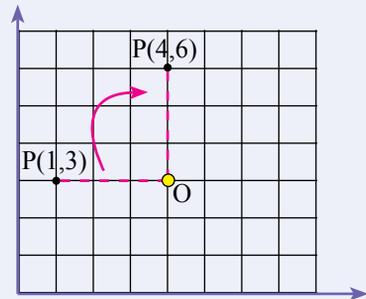


- 4 What are the new coordinates of  $P(1,4)$  after  $P$  is reflected in the line  $AB$ ?
- 5 What are the new coordinates of  $P(1,2)$  after  $P$  is reflected in the line  $AB$ ?
- 6 What are the new coordinates of  $P(2,4)$  after  $P$  is reflected in the line  $AB$ ?

What are the new coordinates of  $P(1,3)$  after  $P$  is rotated  $90^\circ$  clockwise about the point  $O$ ?

$$= \underline{P(4,6)}$$

{Plot the points to work out what the new coordinates will be.}



- 7 What are the new coordinates of  $P(2,3)$  after  $P$  is rotated  $90^\circ$  clockwise about the point  $O$ ?
- 8 What are the new coordinates of  $P(0,3)$  after  $P$  is rotated  $90^\circ$  clockwise about the point  $O$ ?
- 9 What are the new coordinates of  $P(1,3)$  after  $P$  is rotated  $90^\circ$  anti-clockwise about the point  $O$ ?

## Mental Computation

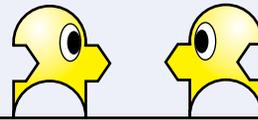
### Exercise 7.6

- 1 Spell transformation.
- 2 Change 0.3 to a fraction.
- 3 Change  $\frac{2}{5}$  to a decimal.
- 4 Translate P(2,3) 2 units to the right and 4 units up.
- 5  $-5 + 3$
- 6 Change  $\frac{8}{5}$  to a mixed numeral.
- 7 Simplify:  $6b - 4b$ .
- 8 What are the four tests for congruent triangles?
- 9 Round 5.2641 to 2 decimal places.
- 10 Simplify:  $x^5 \times x^2$

You need to be a good mental athlete because many everyday problems are solved mentally.

### Exercise 7.7

- 1 Spell congruent.
- 2 Change 0.5 to a fraction.
- 3 Change  $\frac{1}{4}$  to a decimal.
- 4 Translate P(1,3) 3 units to the right and 2 units down.
- 5  $-2 + 3$
- 6 Change  $1\frac{2}{3}$  to an improper fraction.
- 7 Simplify:  $3x + 2x$ .
- 8 What are the four tests for congruent triangles?
- 9 Round 6.3475 to 2 decimal places.
- 10 Simplify:  $x^3 \times x^4$



In just two days from now, tomorrow will be yesterday.

### Exercise 7.8

- 1 Spell reflection.
- 2 Change 0.25 to a fraction.
- 3 Change  $\frac{3}{4}$  to a decimal.
- 4 Translate P(5,3) 3 units to the left and 1 unit down.
- 5  $-4 + 6$
- 6 Change  $\frac{7}{3}$  to an improper fraction.
- 7 Simplify:  $5c - 2c$ .
- 8 What are the four tests for congruent triangles?
- 9 Round 9.2173 to 2 decimal places.
- 10 Simplify:  $x^2 \times x^4$

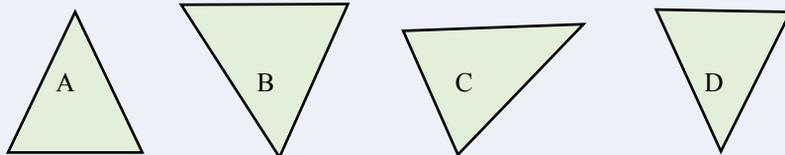
Tomorrow is often the busiest day of the week - Spanish Proverb.



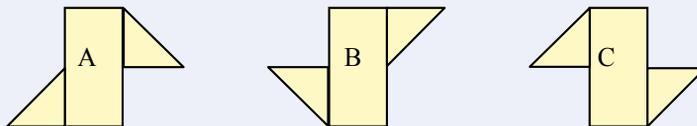
## Exercise 7.9

1 Which pair of the following shapes are congruent?

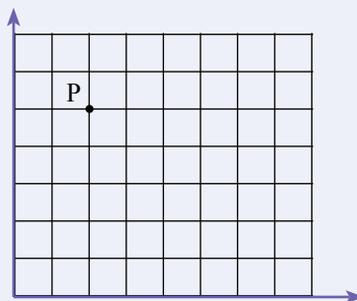
a)



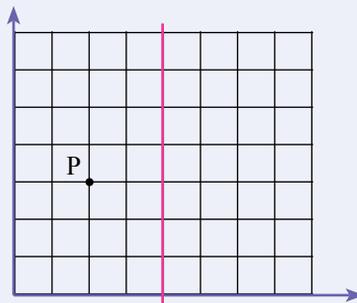
b)



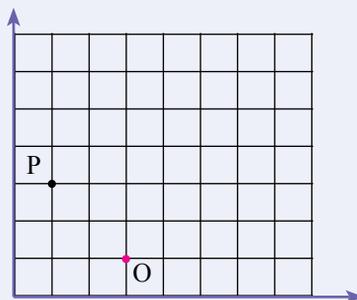
2 The point  $P(2,5)$  is translated 3 units down and 4 units to the right. What are the new coordinates of point P?



3 The point  $P(2,3)$  is reflected in the mirror (the red line). What are the new coordinates of point P?



4 The point  $P(1,3)$  is rotated 90 clockwise about O. What are the new coordinates of point P?



5 SSS, SAS, AAS, and RHS are tests for congruent triangles. What would be the test for congruent circles?

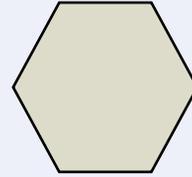
## Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.



### Exercise 7.10

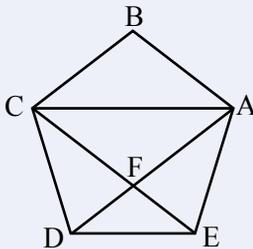
- 1 Divide the regular hexagon, all sides equal, into:
- 2 congruent shapes
  - 3 congruent shapes
  - 4 congruent shapes
  - 6 congruent shapes



- 2 Can you use a single straight line to divide each of the following letters into two congruent shapes?

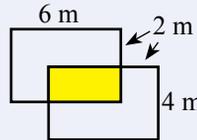
A B C D E F

- 3 Does the diagonal divide the rectangle, the parallelogram, the rhombus, and the kite into two congruent shapes?
- 4 What does the letter F look like after being reflected in a mirror?
- 5 Name three pairs of congruent triangles in the regular pentagon below:

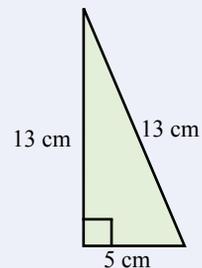
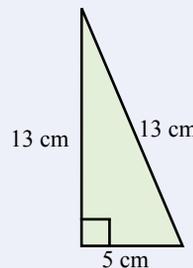
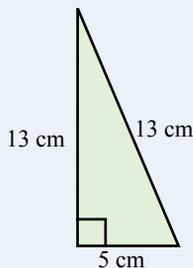
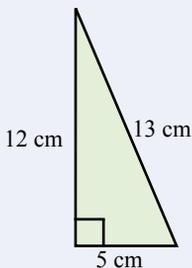


The statement below is true.  
The statement above is false.

- 6 What is the area of the intersection of the two congruent rectangles?



- 7 Rearrange the four congruent triangles to form a square with a hollow square in the middle. What is the area of the middle square?



## Technology

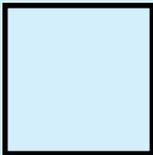
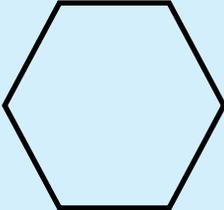
### Technology 7.1 Tessellations

There are numerous tessellation activities and games on the Internet. Experiment with some of these activities.

### Technology 7.2 Tessellations with LOGO

LOGO is a computer programming language created in 1967 for educational use.

- ☞ LOGO is essentially a turtle with a pen on the monitor.
- ☞ The turtle will draw geometric shapes when given movement commands.
- ☞ The LOGO programming language is a powerful programming language.
- ☞ LOGO is considered an ideal introduction to computer programming.

<b>To draw a square:</b> PENDOWN FORWARD 100 LEFT 90 FORWARD 100 LEFT 90 FORWARD 100 LEFT 90 FORWARD 100 LEFT 90 PENUP		<b>To draw a regular hexagon</b> REPEAT 6 [FD 100 LEFT 60]	
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There are a number of free LOGO applications on the Internet and many tutorials, even including tutorials on LOGO and tessellations.

After just a short time learning the language you will be able to start making tessellations with LOGO.

### Technology 7.3 Tessellation with other software

- a) 'Kali' allows you to draw some wonderful tessellations.
- b) 'Tess' also allows you to draw some wonderful tessellations as demonstrated on their website.
- c) 'Tessellation Exploration' all kinds of transformations of basic shapes.
- d) 'Tessellate' an online Javascript tessellation activity.

**Graphic Designers or Commercial Artists** design and produce art and layouts for advertising, magazines, newspapers, books, websites etc.

- Relevant school subjects are English and Mathematics.
- Courses range from Diploma, to Advanced Diplomas, and University Degrees.

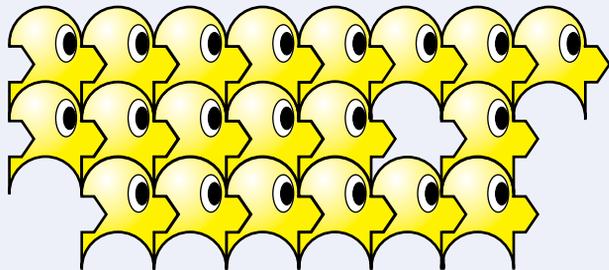
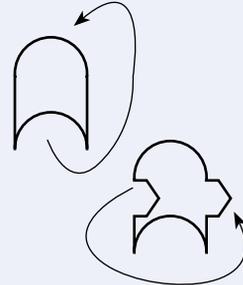
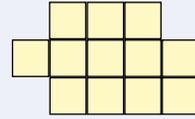
## Investigations

### Investigation 7.1 Make your own tessellation

Tessellations are about designing congruent shapes that fill a space without gaps or overlapping. Tessellations may be formed by joining translated, reflected, and rotated congruent shapes.

The square is a basic shape that will tessellate:

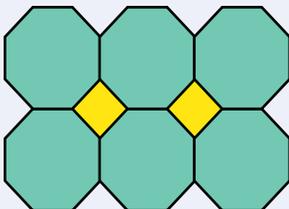
- 1 Select a shape that will tessellate.  
A square is a good shape to start with.
- 2 Cut a piece from the bottom and add it to the top.
- 3 Cut a piece from the left and add it to the right.
- 4 This shape, based on a square, should now tessellate.  
Decorate the piece.
- 5 Make multiple copies and tessellate - use a photocopier or tessellation software.



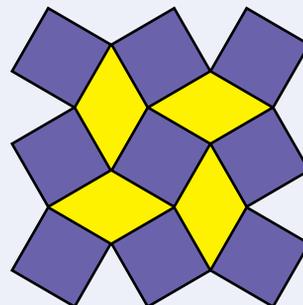
### Investigation 7.2 Regular polygon tessellation

Experiment with tessellations of combinations of polygons. Try combinations of shapes such as triangles, squares, parallelograms, pentagons, hexagons etc:

Squares and Octagons



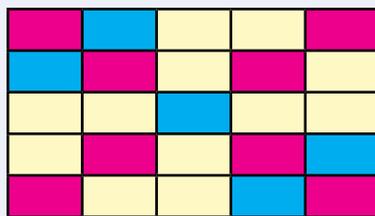
Squares and Parallelograms



## A Couple of Puzzles

### Exercise 7.11

1 What percentage of the rug is blue?  
What percentage of the rug is yellow?



2 Hume exercises 30 mins each day.  
How many hours does Hume exercise  
in a 30 day month?

3 I have four daughters.  
Every daughter has a brother.  
How many children?

I am a nobody.  
Nobody is perfect.  
Therefore I am perfect.

4 A is three times the age of B. In ten years time, A will be twice the age of B.  
How old are A and B?



This is a well known puzzle.  
Join all 9 dots using only 4 straight lines.  
Do not lift your pencil from the page.

## A Game

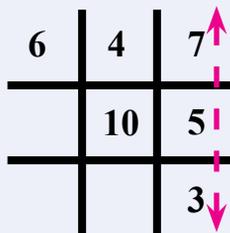
**Odds and Evens** is naughts and crosses played with the numbers.

2, 4, 6, 8, 10    1, 3, 5, 7, 9

1 One player uses the odd numbers,  
the other player uses the even numbers.

2 When you use a number cross it off  
because it can't be used more than once.

3 The winner is the first to get a sum of 15  
either horizontally, vertically, or diagonally.



## A Sweet Trick

- 1 Ask your audience to choose their favourite number from 1 to 9.
- 2 Write the number three times.
- 3 Sum the digits of the number.
- 4 Multiply the answer by 37

All the flowers of tomorrow  
are in the seeds of yesterday.

$$7$$

$$777$$

$$7+7+7 = 21$$

$$21 \times 37 = 777$$

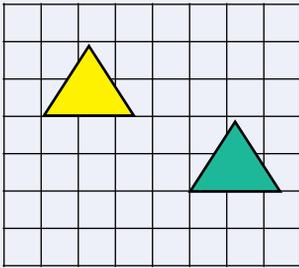


# Chapter Review 1

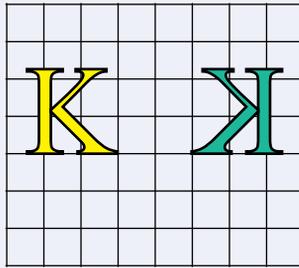
## Exercise 7.12

1 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

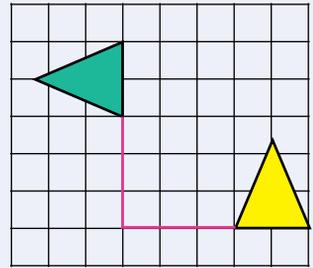
a)



b)

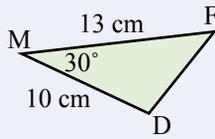
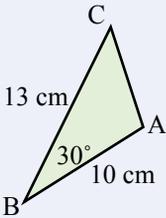


c)

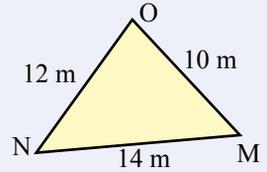
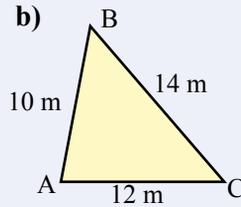


2 Use the tests for congruence to test whether the following pairs of triangles are congruent:

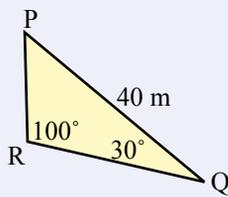
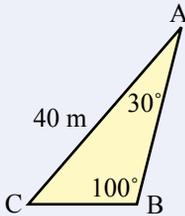
a)



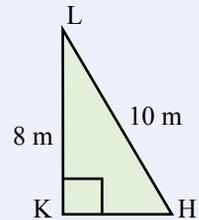
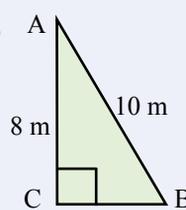
b)



c)

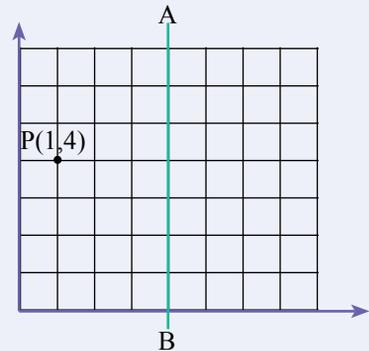
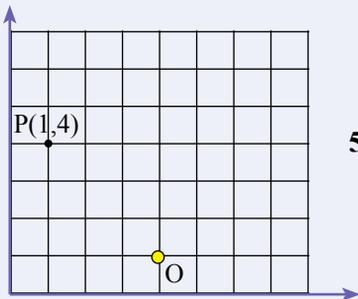


d)



3 What are the new coordinates of  $P(3,5)$  after  $P$  is translated 3 units left and 3 units up?

4 What are the new coordinates of  $P(1,4)$  after  $P$  is reflected in the line  $AB$ ?



5 What are the new coordinates of  $P(1,4)$  after  $P$  is rotated  $90^\circ$  clockwise about the point  $O$ ?

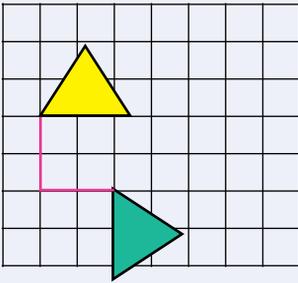
I told you to be home by a quarter of twelve.  
A quarter of twelve is 3.

## Chapter Review 2

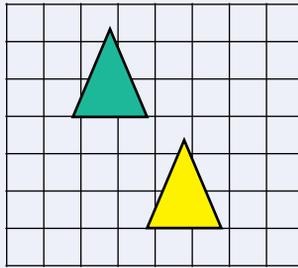
### Exercise 7.13

1 Describe the transformation (translation, reflection, or rotation) to produce the congruent shape from the original:

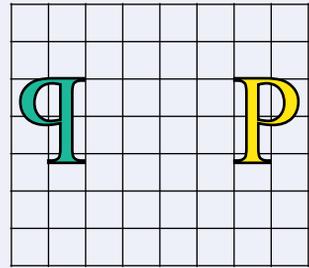
a)



b)

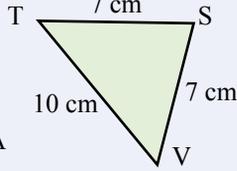
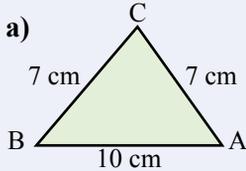


c)

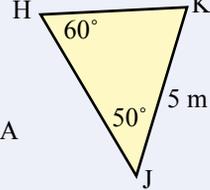
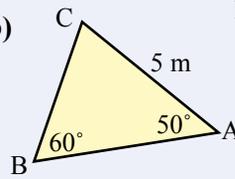


2 Use the tests for congruence to test whether the following pairs of triangles are congruent:

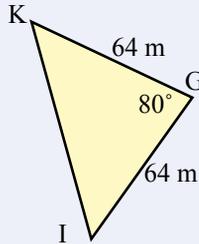
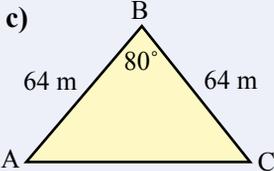
a)



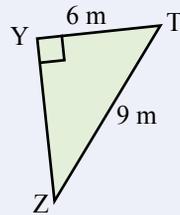
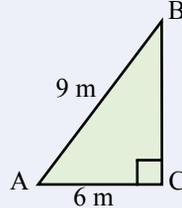
b)



c)

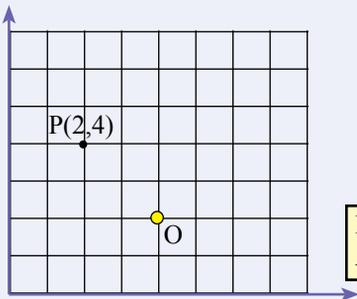
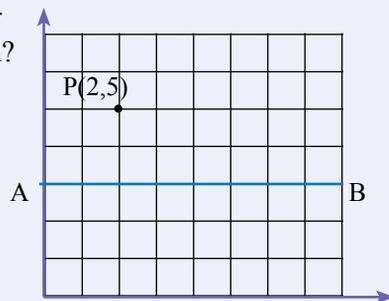


d)



3 What are the new coordinates of P(2,6) after P is translated 4 units right and 5 units down?

4 What are the new coordinates of P(2,5) after P is reflected in the line AB?



5 What are the new coordinates of P(2,4) after P is rotated  $45^\circ$  clockwise about the point O?

My short-term memory is not as sharp as it used to be.  
Also, my short-term memory is not as sharp as it used to be.